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GEOMETRIC CHARACTERISTICS OF FRACTURE-ASSOCIATED SPACE AND CRACK PROPAGATION IN A MATERIAL

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It is shown that fracture can be treated as a process occurring in the Finsler space. The use of the Finsler space allows one to construct a delaminated manifold whose characteristics are related to the defect structure of the medium. A method of determining the fractal dimension of fracture is developed using the concept of crack propagation along geodesics.

Key words: *fracture, thickness, defect structure, Finsler space.*

1. Geometric Representations in the Fracture Theory. Processes occurring in the vicinity of a propagating crack are a complex combination of the processes of elastic and plastic deformation of the material and accumulation of microscopic cracks, which lead to the formation of a macrocrack. The interaction of the stress field of the propagating crack with the internal-stress field of the basic material and defect field (e.g., dislocations, disclinations, and point defects) is difficult to describe because of the complexity of the physical processes and mathematical apparatus that adequately governs these processes.

In most studies dealing with prediction of crack propagation, the "global" crack propagation and "global" trajectories are considered; however, this averaging approach does not allow one to introduce fractal characteristics, which are local in nature. The "local" approach to determining the crack trajectory can be developed using the variational principle [1].

The stress fields used to calculate the trajectories in the "local" approach of the theory of cracks should be determined on the basis of the microstructure of a material to fail. In principle, this is possible in the continual theory of dislocations. The basic idea of the continual theory developed by B. A. Bilby, E. Kröner, A. M. Kosevich, and I. A. Kunin is to find a relation between defects inherent in a real solid and geometry (metric properties) of the medium. The geometric approach adequately describes the imperfection of crystals. However, the relation between the metric properties of the medium and the processes of plastic deformation and fracture (crack propagation) is not yet clearly understood.

In the present paper, the effect of the metric properties of the medium (metric tensor) on the crack trajectory is studied. The fractality of the fracture process (crack propagation) is related to the specific geometric features of the space whose properties depend on the structure of the failing material.

2. Trajectory of a Crack. Using the optical-mechanical analogy, Miklashevich and Chigarev [2, 3] derived the following crack-trajectory variational equation for the simplest crack model of the Barenblatt–Dugdale type:

$$\frac{\partial Q}{\partial y}\,\frac{\sqrt{1+y'^2}}{Q^2}+\frac{d}{dx}\,\frac{y'}{Q\sqrt{1+y'^2}}=0,\quad Q=(\sigma_{ij}n_iu_j)^{-1},\quad y'=\frac{dy}{dx}. \tag{1}$$

Here σ_{ij} are the stresses at the crack edges, n_i is the direction cosine of the external normal to the crack surface, and u_j is the displacement of the crack edges. Equation (1) is obtained for an ideal medium without allowance for the real microstructure of the material. Self-consistency of Eq. (1) [propagation of the crack at the $(i-1)$th

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step affects the direction of its growth at the ith step] necessary to describe the fracture surface [4] is ensured by step-by-step varyation of the fracture energy in the near vicinity of the crack. We introduce the notation

$$f_1(x,y) = \frac{\partial \ln Q(x,y)}{\partial x}, \qquad f_2(x,y) = \frac{\partial \ln Q(x,y)}{\partial y}$$

and write Eq. (1) as [3]

$$y'' - y' f_1(x,y)(1+y'^2) + f_2(x,y)(1+y'^2)^2 = 0. \tag{2}$$

Since Eqs. (1) and (2) are derived from the variational principle, they are equations of geodesics of the fracture-energy release and are determined in the Euclidean space. This agrees with the well-known statement that real cracks propagate along geodesics [5]. In Eq. (2), the expression $1 + y'^2$ is the arc-length element for the two-dimensional Euclidean space. The use of the Euclidean metric allows one to obtain realistic equations of the trajectories in a few cases [1, 6] for ideal materials. For real media with a microstructure (defects), Eq. (1) should be generalized to wider subspace classes. The reason is that, for media with an internal structure, the processes occurring in the vicinity of the crack tip are very complex and cannot be described as a direct sum of operators of plastic and elastic deformation [7]. To take into account the defect structure of a material, it is necessary to introduce all three nonzero Kartan curvature tensors [8–10]; therefore, the space whose properties depend on the structure of the failing material should have a more general character compared to the Euclidean and Riemannian spaces [10–12].

Generalized spaces can be constructed in two ways [12, 13]: 1) introduction of the metric tensor (or metric function) into the manifold; 2) introduction of the connectivity coefficients into the manifold. In studying the fracture processes, the second way is preferable because here the defect structure determines the geometric structure of the space. For the Riemannian space, it is impossible to introduce the metric and connectivity independently since the connectivity coefficients of the Riemannian space are uniquely related to the metric [12]:

$$\Gamma^k_{ij} = \frac{1}{2}\, g^{kl}\Big(\frac{\partial g_{li}}{\partial x^j} + \frac{\partial g_{lj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l}\Big). \tag{3}$$

At the same time, the general affine geometries allows one to consider the connectivity deformations necessary for obtaining the complete set of nontrivial affine-metric characteristics in accordance with chosen internal invariants of strain and fracture [13]. In constructing the set of determining parameters (and corresponding connectivity deformations), it should be borne in mind that an adequate description of crack-propagation dynamics requires introduction of velocities as independent parameters into the governing equations of the process [7, 14].

We consider the question of determining the crack trajectory with allowance for the real structure of the material. In the case of an arbitrary generalized space, the equation of a geodesic can be written as a function of the arc length s of the curve, current coordinates x^μ, and connectivity coefficients of the space $\Gamma^\mu_{i\lambda}$:

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\sigma\lambda}\,\frac{dx^\mu}{ds}\,\frac{dx^\lambda}{ds} = 0. \tag{4}$$

Determination of the crack trajectory reduces to determination of the connectivity coefficients of the generalized space.

3. Geometric Characteristics of Solids with Defects. In studying defects in the general form, correspondence between a solid body (continuum) and a certain manifold M_n is established. Depending on interpretation of defect characteristics, the manifold can be of one or another geometric character ranging from theories of limited application (in which the curvature tensor is $R^l_{ij,k} = 0$) to metric affine-connected manifolds of general form [9]. (Below, the unit vectors of the space are denoted by Latin subscripts.) For an arbitrary manifold M_n, the Finsler space [15] is one of the simplest generalized spaces whose metric admits the existence of all three nonzero curvature tensors and which is used to describe media with a microstructure. We choose this space since, for this space, the Hamilton function of the system $H(x,y)$ and the metric function $F(x,\dot{x})$ are related by the standard canonical equations

$$\frac{\partial F(x,\dot{x})}{\partial x^i} = -\frac{\partial H(x,y)}{\partial x^i}, \tag{5}$$

where $\dot{x} = dx^i/dt$ and x^i, $\dot{x}^i$, and y_i are independent variables. Introduction of velocities as independent variables makes it possible to study the dynamic processes without additional assumptions. We also introduce the notation $f(x^i, x^j, \ldots) = f(x)$.

Since the Hamiltonian has the character of the bearing surface for the indicatrix, the functions H and F are dual functions, the metric function $F(x, \dot{x})$ is the Lagrangian of the system [7, 15], and Eqs. (5) are the standard Hamilton–Jacobi equations. Spaces of this type are also used to model plastic deformation.

Introduction of a metric function of the form $F(x, \dot{x})$ determines a delaminated manifold in which $\dot{x}^i$ define the linear vector space T_n tangent to the basic manifold M_n and are contravariant vectors of the space T_n. In this case, the tangency point is a point of the manifold $P(x^i)$. For the Finsler space, we introduce the metric tensor

$$g_{ij}(x, \dot{x}) = \frac{\partial^2 F^2(x, \dot{x})}{\partial \dot{x}^i \partial \dot{x}^j}. \tag{6}$$

For each arbitrary contravariant vector $\dot{x}^i \in T_n$, one can find the covariant vector y_i of the dual tangent space:

$$y_i = g_{ij}(x, \dot{x})\dot{x}^j. \tag{7}$$

The Riemannian space is a particular case of the Finsler space, for which $g_{ij} \neq g_{ij}(\dot{x}^i)$, i.e., the metric tensor is direction-independent. Using metric (6), one can formulate analogs of the known geometry theorems and Riemannian geometry [15]. For the Riemannian space, the tangent space is the Euclidean space, and the Riemannian space can be considered as a locally Euclidean space. Locally, the Finsler space is the Minkowski space.

4. Effect of the Defect Structure on Crack Propagation. It is well known that processes of elastic deformation in an ideal crystal can be geometrically interpreted as processes in the Riemannian space with a direction-independent metric. In this metric, the elementary length can be written as

$$dl^{(r)} = \sqrt{g_{ij}^{(r)} \frac{dx^{i(r)}}{dt} \frac{dx^{j(r)}}{dt}}.$$

Here, the superscript r at the metric tensor g_{ij} denotes the Riemannian space, x^i and x^j are the current coordinates of the point, and t is the natural parameter of the curve; the indices run from 0 to n (n is the dimension of the space considered in the problem). In analyzing the propagation of a crack, the crack length is usually used as the natural parameter.

4.1. *Effect of the Microstructure on the Metric Properties of a Continuum.* Since each type of defects is characterized by an additional geometric parameter of the space, for example, curvature S, torsion R, and segmentary curvature K (the first, second, and third Kartan curvature tensors), in the case of a medium with a microstructure, the metric depends not only on the position of the cocurrent coordinate system (macroscopic state) but also on the vector field associated with defects (microscopic state). In this case, the microstructure (distribution and nature of defects) is generally independent of the macroscopic state, i.e., generation and evolution of these states can occur in independent ways. For example, point defects can result from irradiation, which leaves the strain state almost unchanged. Moreover, as was mentioned above, the physical meaning of the problem implies that the geometric properties depend also on the direction in this space, determining characteristics of the space, and their velocities [4, 11]. Therefore, in the general case, we have $g_{ij} = g_{ij}(S, R, K, x^j, \dot{x}^j)$, which allows one to determine the metric tensor for the Finsler space as a function of connectivity coefficients from the differential equation

$$\Big(\frac{\partial \Gamma^{*i}_{k.l}}{\partial x^t} - \frac{\partial \Gamma^{*i}_{k.l}}{\partial \dot{x}^s}\frac{\partial G^s}{\partial \dot{x}^t} + \Gamma^{*i}_{q.t}\Gamma^{*q}_{k.l}\Big) = g^{ij}\Big(\frac{\partial \Gamma^{*}_{kjl}}{\partial x^t} - \frac{\partial \Gamma^{*}_{kjl}}{\partial \dot{x}^s}\frac{\partial G^s}{\partial \dot{x}^t} - \Gamma^{*q}_{j.t}\Gamma^{*}_{kql}\Big). \tag{8}$$

Here $\Gamma^*_{kjl} = \Gamma^*_{kjl}(S, R, K, x^i, \dot{x}^i)$ are the symmetric connectivity coefficients of the Finsler space (which differ from the connectivity coefficients of the Riemannian space in the general case). In view of their cumbersome expressions, the connectivity coefficients are not given here. We consider the main properties of the dependence of the metric function on the velocity of determining parameters. In Eq. (8), the quantity G^i describes nonmetricity of connectivity and arises owing to the fact that the Finsler metric depends not only on the position of the cocurrent coordinate system but also on the additional vector field ξ^l (field associated with defects). This quantity can be found from the equations for the field derivatives [15] or obtained from the introduced connectivity deformation [13]. It is worth noting that, in the general case, G^i does not depend on the metric tensor, which ensures the influence of material imperfections on the fracture processes regardless of the geometry. Thus,

$$\Gamma^i_{k.j}\dot{x}^k = \frac{\partial G^i}{\partial \dot{x}^j}, \qquad \Gamma^{*i}_{k.j} = \Gamma^i_{k.j} - C^i_{k.h}\Gamma^h_{r.j}\dot{x}^r.$$

Here $C^i_{k.j}$ is the characteristic tensor of the Finsler geometry, related to the variation of the metric tensor along the chosen directions (Kartan torsion tensor [7]). The presence of torsion in the Finsler connectivity is due to the effect of the defect structure on the ideal continuum:

$$C_{kji}(x, \dot{x}) = \frac{1}{2}\frac{\partial g_{ij}}{\partial \dot{x}^k} = \frac{1}{4}\frac{\partial^3 F^2(x, \dot{x})}{\partial \dot{x}^k \partial \dot{x}^j \partial \dot{x}^i}. \tag{9}$$

Raising and lowering of indices is performed using the metric tensor determined by Eq. (6). It should be noted that the torsion [tensor (9)] can be specified in different ways. In some cases, the first Kartan curvature tensor [13]

$$S^j_{i.kl} = A^j_{k.r}A^r_{i.l} - A^j_{r.l}A^r_{i.k},$$

where $A^j_{i.k} = F(x, \dot{x})C^j_{i.k}$, is used. Since the length is measured in the real Euclidean space, the "excess" variables of the Finsler space are latent parameters, and their projection onto the tangent Euclidean space determines the fractal character of the surface. This corresponds to introduction of three curvature tensors as additional parameters of solid mechanics [8, 11].

We note that, in the affine-connectivity space, our interest is with geodesics and, hence, we can consider the torsion-free Finsler space as a crack-propagation space. The reason is that the connectivity object $\Gamma^\mu_{\sigma\lambda}$ determines the same geodesic in this manifold as the torsion-free connectivity object $\tilde{\Gamma}^\mu_{\sigma\lambda}$ obtained by its symmetrization. Therefore, we can confine our attention to the one-shape Finsler spaces with the Bervald–Moor metric. For these spaces, the metric is constructed on the basis of an arbitrary Minkowski metric. In this case, the metric tensor of the space has the form

$$g_{ij}(x^m, \dot{x}^m) = S^A_m S^B_i g_{AB}(\dot{x}^D(x^m, \dot{x}^m)),$$

where S^A_m is the global reference field of the C^3 class, and the starting metric of the Minkowski space

$$g_{AB} = \frac{1}{2}\frac{\partial^2 F^2_M}{\partial \dot{x}^A \partial \dot{x}^B}$$

depends on x only through $\dot{x}^D$.

4.2. *Crack Trajectory in a Medium with Microdefects.* In 1966, R. Atkinson studied the crack-dislocation interaction in two-dimensional formulation using the classical theory of anisotropic media. Atkinson's solution was based on determining the interaction forces between the dislocation and its image, which appears at the free surface of the crack. Later, B. A. Bilby, A. H. Cottrell, and K. H. Swinden improved this theory by taking into account dislocation-field screening. The principal difficulty in using both the Bilby–Cottrell–Swinden theory and the classical theory of cracks is the presence of singular solutions in the dislocation core and crack tip, which does not allow one to use the theory of elasticity. Despite the series of generalizations in which the singularity problem was solved, the correctness of these solutions is doubtful. These solutions contradict the experimentally established fact that there are no defects in the neighborhood of the free surface of a crack [16, 17]. Inasmuch as defects are absent on the surface, the object that produces images is also absent.

This contradiction calls for the development of new approaches to study the crack-defect interaction, based, for example, on the Lagrangian formalism of the continual theory of defects [18]. The geometric theory of the crack-defect interaction also holds much promise [19]. The reason is that, from the physical viewpoint, the variational problem of crack propagation (determination of the optimal crack-propagation trajectory) can be considered as the formation and disappearance of virtual free surfaces in the bulk of the material. The energy of these processes is determined by the metric properties of the continuum in the region where the virtual surfaces are formed. Since the energy, like other invariant factors, depends on the defect structure, the metric properties (6) are functions of the defect structure and the geometric parameters are functions of the energy related to the defect structure [7].

Since the defect structure is determined locally, the Finsler space is decomposed into two three-dimensional Euclidean spaces

$$B_A = B \times M \subset E^3 \times E^3,$$

where B is the object (body) considered and M is the microstructure in the reference configuration; the multiplication sign denotes the Cartesian product of the spaces. This representation corresponds to the well-known description of bodies with microstructures by means of the Cosserat media (position and director vectors). This decomposition is attributed to the fact that the action of the defect field occurs in the tangential space, whereas the action of the macrostructure occurs in the Euclidean space related kinematically to the initial undeformed structure. In this case, the metric function $F(x, \dot{x})$ can be written as $F(x, y)$ ($x = x_i$ are the macroscopic coordinates of the point and $x = y^i$ are the microscopic coordinates). The metric tensor (6) has the form

$$g^{ij}(x, y) = \frac{\partial^2 H^2(x, y)}{\partial y_i \partial y_j}. \tag{10}$$

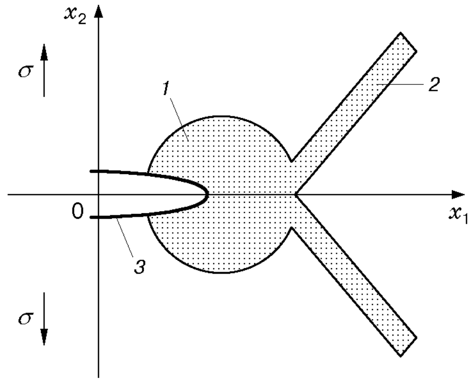

Fig. 1. Region of plastic deformation in the vicinity of the crack tip: plastic-flow zone (1), slip band (2), and crack (3).

As in spaces with the Bervald–Moor metric, the metric tensor depends on x through y only. Moreover, it can be assumed that the geometric characteristics of these spaces in the neighborhood of the tangency point are small.

We consider propagation of a crack in a medium. We take into account the relation between crack propagation and evolution of the defect structure and ignore other processes of energy dissipation. In the simplest case, it is sufficient to consider the development of defects in the region of plastic deformation caused by crack growth. It is known that, as the crack propagates, plastic deformation occurs in the vicinity of the crack tip and in the region of plastic-shear localization (slip bands, etc.) (see Fig. 1). In the two-dimensional case, the region of deformations initiated by the crack can be regarded as superposition of a circular region of radius r whose center lies at the crack tip and rectilinear segments of slip bands. It is worth noting that the exact shape of the plastic-deformation zone is of no importance for the algorithm proposed. Since the space is decomposed into vertical and horizontal subspaces, the total energy concentrated in the dislocation field is a function of microscopic coordinates and is located in the tangent space. This means that, at this point of the continuum, the crack "feels" only a certain part of the defect field of the material, which plays an important role in energy variation (precisely this variation is responsible for the crack-growth direction). We assume that $E \sim \exp(-\mu\sqrt{y_1^2+y_2^2})$. At the same time, the energy should be a function of macroscopic coordinates. The reason is that the "distance" between the point of the basic and tangent spaces is a function of their radius vectors; determination of this "distance" is a separate problem. We assume that, for a circular region, the dislocation-distribution function depends on macroscopic parameters (coordinates x^1 and x^2) and is uniform inside the region of radius r; outside this region, the influence of the defect field decreases exponentially. We introduce the notation $q = r^2 - ((x^1)^2 + (x^2)^2)$. The macroscopic dependence can be written as

$$n_1(x,y) = n_{01}\{\theta(q) + \theta(-q)\exp[-\lambda_1((x^1)^2 + (x^2)^2)]\}, \tag{11}$$

where the characteristic of the medium $\lambda_1 > 0$ is a coefficient that takes into account the defect-continuum interaction (for example, deceleration forces and processes of generation and annihilation of defects) and θ is the Heaviside function. With allowance for Eq. (11), the total energy concentrated in the defects of the circular region with the macroscopic coordinates x^1 and x^2 can be written as

$$E_1 = E_0\int n_1(x,y)\exp(-\mu\sqrt{y_1^2+y_2^2})\,dS. \tag{12}$$

Here E_0 is the elastic energy of unit dislocation, dS is the element of the tangent-space area, which is a region where the defect distribution affects the energy of defects at the given tangency point of the spaces. Since the material is heterogeneous in the general case, we consider an elliptic elementary area with semiaxes a and b whose orientation depends on the heterogeneity. In this case, it follows from Eq. (12) that

$$E_1 = E_0\pi ab\exp(-\mu\sqrt{y_1^2+y_2^2})\{\theta(q) + \theta(-q)\exp[-\lambda_1((x^1)^2 + (x^2)^2)]\}. \tag{13}$$

We consider an infinitely thin slip band. Starting from similar reasoning, we assume that the density of defects in the band has the form

$$\begin{aligned} n_2(x,y) &= n_{02}\delta(\pm kx_1 - B - x_2)\exp[-\lambda_2((x^1)^2 + (x^2)^2)] \\ &= n_{02}\delta(\pm t - x^2)\exp[-\lambda_2((x^1)^2 + (x^2)^2)\,], \end{aligned} \tag{14}$$

where n_{02} is the initial density of the defect distribution in the slip band, δ is the Dirac delta function, $k = \tan\alpha$ (α is the slope angle of the slip band), B is the coordinate of the slip-band origin, and $\lambda_2 > 0$. The plus and minus signs in the argument of the delta function refer to the slip bands in the upper and lower halfplanes, respectively. We denote the radius-vector of microstates by $\rho^2 = y_1^2 + y_2^2$ and the radius-vector of macrostates by $\tilde{\rho}^2 = (x^1)^2 + (x^2)^2$. Assuming that interactions in the tangent space are the same for the circular region and band, with allowance for (13) and (14), we obtain the total energy

$$E = E_1 + E_2 = \pi a^2 b^2 \,\mathrm{e}^{-\mu\rho}[\theta(r^2 - \tilde{\rho}^2) + \theta(\tilde{\rho}^2 - r^2)\,\mathrm{e}^{-\lambda_1\tilde{\rho}} + \delta(\pm kx^1 + b - x^2)\,\mathrm{e}^{-\lambda_2\tilde{\rho}}]. \tag{15}$$

Since $a = a(y_1, y_2)$ and $b = b(y_1, y_2)$, we can calculate the components of the metric tensor of the Finsler space g^{ij} using (10) and (15). In view of cumbersome expressions obtained, we give only the component g^{11}:

$$g^{11} = \Big[\Big(\frac{\partial a}{\partial y_1}\Big)^2 b^2 + \Big(\frac{\partial b}{\partial y_1}\Big)^2 a^2 + ab\Big(b\,\frac{\partial^2 a}{\partial y_1^2} + a\,\frac{\partial^2 b}{\partial y_1^2}\Big) + 4ab\,\frac{\partial a}{\partial y_1}\,\frac{\partial b}{\partial y_1}$$

$$- 3ab\mu\,\frac{y_1}{\rho}\,A - a^2 b^2 \mu\,\frac{y_2^2}{\rho^3} + a^2 b^2 \mu^2\,\frac{y_1}{\rho^2}\Big]\,2\,\mathrm{e}^{-2\mu\rho}$$

$$\times \Big[\theta(r^2 - \tilde{\rho}^2) + \theta(\tilde{\rho}^2 - r^2)\,\mathrm{e}^{-\lambda_1\tilde{\rho}} + \delta(\pm kx^1 + b - x^2)\,\mathrm{e}^{-\lambda_2\tilde{\rho}}\Big]. \tag{16}$$

Here $A = b\partial a/\partial y_1 + a\partial b/\partial y_1$. Expression (16) was obtained with allowance for $\partial\rho/\partial y_1 = y_1/\rho$ and $\partial^2\rho/\partial y_1^2 = y_2^2/\rho^3$.

Using the expression for the metric tensor of a medium with defects (10) and the concept of crack propagation along a geodesic, we write Eq. (4) in the form

$$\frac{dy_i}{ds} - \gamma_{ihk}(x, x')x'^h x'^k = 0. \tag{17}$$

Here s is the Finsler arc-length parameter, the vectors $x'^i = \dot{x}^i(dt/ds)$ and y_i are related by (7), and $\gamma_{ihk}(x, x')$ are the Christoffel symbols of the first kind, determined in the same manner as in the Riemannian geometry.

Equation (17) gives physical substantiation of the fractal character of crack propagation. Determining the fractal dimension of a crack as the ratio of the trajectory length in a real crystal to the trajectory length in an ideal crystal (i.e., the ratio of the trajectory lengths in the Finsler and Riemanian spaces), for identical parametrization of the curves, we obtain

$$D = \frac{dl}{dl^r} = \sqrt{g_{is}\,\frac{dx^i}{dt}\,\frac{dx^s}{dt}} \Big/ \sqrt{g_{is}^r\,\frac{dx^{i(r)}}{dt}\,\frac{dx^{s(r)}}{dt}} = \frac{\sqrt{g_{is}\,dx^i\,dx^s}}{\sqrt{g_{is}^{(r)}\,dx^{i(r)}\,dx^{s(r)}}}. \tag{18}$$

Since the symmetric connectivity coefficients are functions of state, the fractal dimension of a crack is also a function of state.

Conclusions. Using the geometry of the Finsler space to describe deformation of bodies with a defect structure, one obtains a physically substantiated basis for analysis of the interaction between the geometric structure of an ideal material and the geometric structure of a system of defects without involving any additional assumptions. From the viewpoint of delaminated manifolds generated by the Finsler metric, horizontal delamination corresponds to an ideal continuum and vertical delamination to defects of various types.

The existence of the vertical components of the manifold allows one to reveal the physical reasons for the fractal character of crack propagation. In accordance with experimental data and numerical models, the fractal dimension of fracture [see Eq. (18)] depends on both the properties of real crystals and failure conditions.

For modes of prescribed failure (where the material should fail in a given region and, if possible, along a specified trajectory), the trajectory of crack propagation is determined by Eq. (17). In this case, one can obtain the desired characteristics of the crack trajectory by specifying the defect distribution and controlling this distribution by technological means. In the process, the dislocation-distribution function is a free parameter which has no effect on surface fractality but affects the mechanical parameters of the material (for example, Lamé parameters).

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